

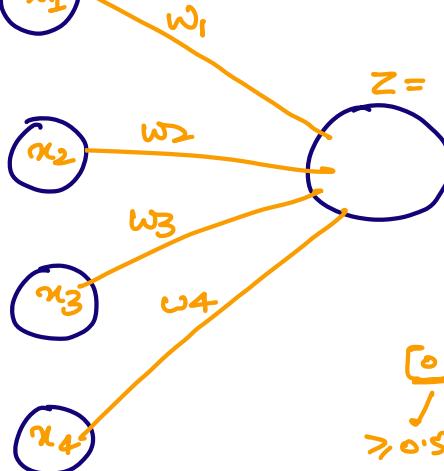
Neural Architecture

1 layer network

logistic function

Binary Classification

$$z = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + b$$



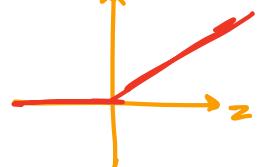
$$\sigma(z) \hat{y}$$

Activation fn (Sigmoid)

$$[0 - 1]$$

≥ 0.5 Use 1
 < 0.5 Use 0

Rule: $\sigma(z)$

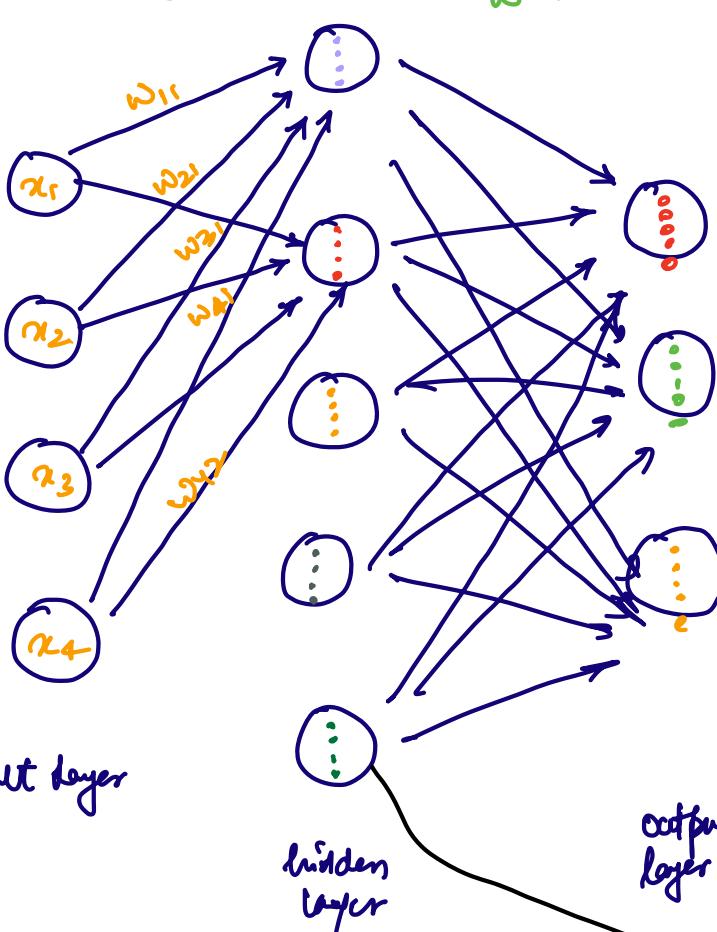


$$\begin{cases} \sigma(z) = z & z \geq 0 \\ \sigma(z) = 0 & z \leq 0 \end{cases}$$

2 layer network

$$w^{[1]}$$

$$w^{[2]}$$



output layer

hidden layer

Input layer

$$w^{[1]} =$$

$$\begin{bmatrix} w_{11} & w_{12} & \dots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ w_{21} & w_{22} & \dots & \vdots & \vdots \end{bmatrix}$$

w_{21} w_{32}
 w_{41} w_{42}

\vdots \vdots

4×5
 no. of neurons in i/p layer
 no. of neurons in hidden layer

$$w^{[1]} = \begin{bmatrix} | & | & | & | & | \\ w_1^{[1]} & w_2^{[1]} & w_3^{[1]} & w_4^{[1]} & w_5^{[1]} \\ | & | & | & | & | \end{bmatrix}$$

weights associated with 5th neuron of 1st layer

$$w^{[2]} = \begin{bmatrix} 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{5 \times 3}$$

Output Layer:

Dog		$\rightarrow 0.6$
Cat		$\rightarrow 0.1$
Horse		$\rightarrow 0.3$

Dog class

The diagram shows the softmax calculation for the Dog class output. It starts with three values: 10, 20, and 30. These are divided by 10 to get e^{10} , e^{20} , and e^{30} respectively. These values are then summed to get the denominator $e^{10} + e^{20} + e^{30}$. The first term $e^{10}/(e^{10} + e^{20} + e^{30})$ is shown as the output for the Dog class.

$$\frac{e^{10}}{e^{10} + e^{20} + e^{30}} = \text{Dog class output}$$

$$\frac{e^{20}}{e^{10} + e^{20} + e^{30}} = \text{Cat output}$$

$$\frac{e^{10}}{e^{10} + e^2 + e^0} = 0$$

No. of parameters:

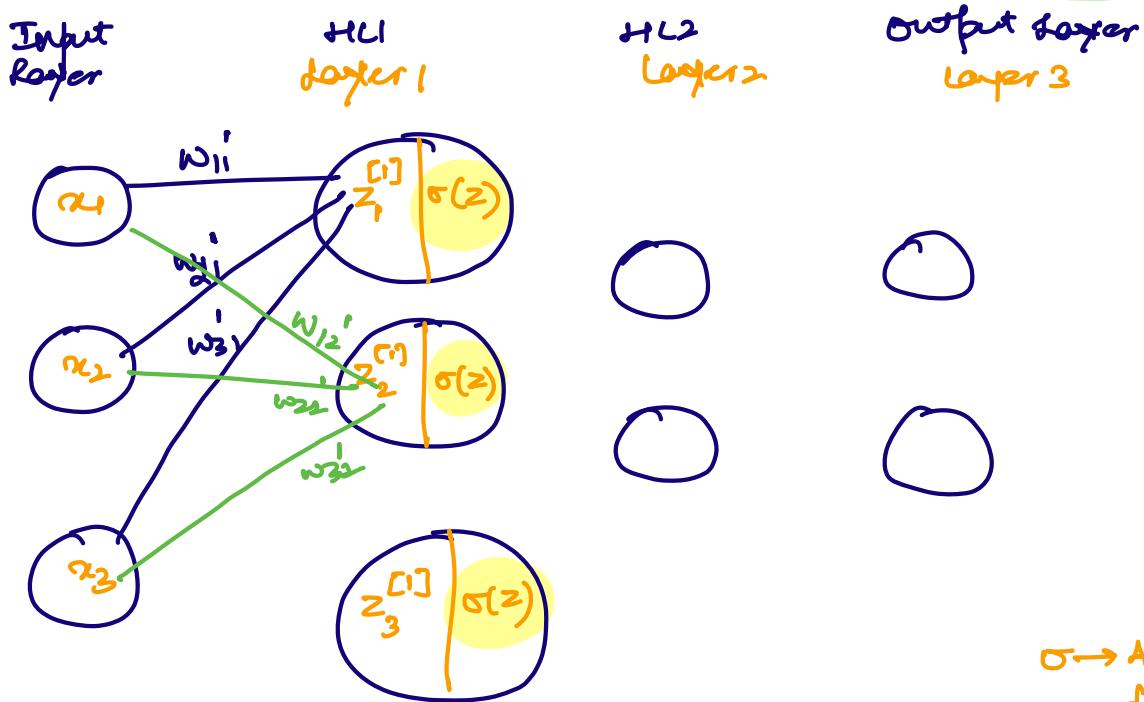
$$\begin{aligned}
 & \xrightarrow{\omega^{[1]}} \xrightarrow{\omega^{[2]}} \xrightarrow{b^{[1]}} \xrightarrow{b^{[2]}} \\
 & 4 \times 5 + 5 \times 3 + 5 + 3 \\
 & = 20 + 15 + 5 + 3 \\
 & = 43
 \end{aligned}$$

LR:

$$w_0 x_0 + w_1 x_1 + w_2 x_2 + \dots + w_n x_n$$

$$w_0 = 1 \rightarrow (n+1)$$

3 layer:



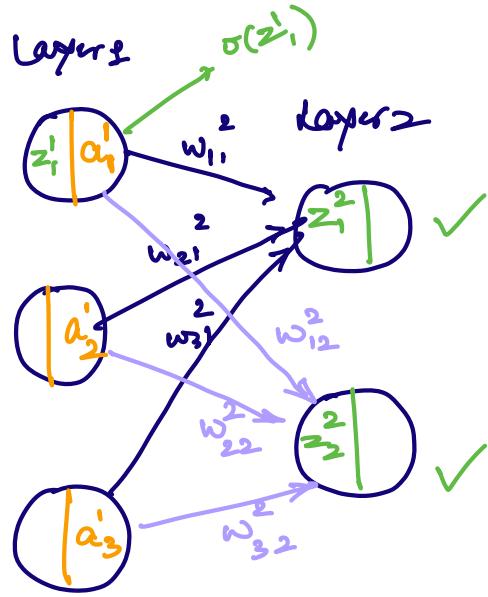
$\sigma \rightarrow$ Activation
 f_{act}

$$z_1^{[1]} = x_1 * w_{11}^1 + x_2 * w_{21}^1 + x_3 * w_{31}^1 + b_1^{[1]} = \underline{\text{Result}}$$

$$z_2^{[1]} = x_1 * w_{12}^1 + x_2 * w_{22}^1 + x_3 * w_{32}^1 + b_2^{[1]}$$

$$z_j^{[l]} = \sum_{i=1}^n w_{ij}^{[l]} x_i + b_j^{[l]}$$

$n = \text{no. of features}$



$$z''_1 = w_{11}^2 * a'_1 + w_{21}^2 * a'_2 + w_{31}^2 * a'_3 + b_1^2$$

$$z''_2 = w_{12}^2 * a'_1 + w_{22}^2 * a'_2 + w_{32}^2 * a'_3 + b_2^2$$

$$z_j^{[l]} = \sum_i w_{ij}^{[l]} * a_i^{[l-1]} + b_j^{[l]}$$

$i \in \text{all neurons from } l-1 \text{ layer}$

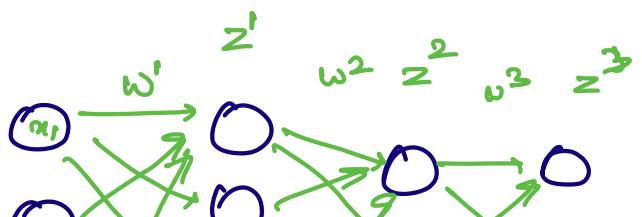
$a_i^{[l-1]} = \begin{bmatrix} - \\ - \\ - \end{bmatrix}$

Vektor:

$$\begin{aligned} & \text{Matrix: } \begin{bmatrix} 3 \times 2 & 3 \times 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a_i^{[l-1]} = \begin{bmatrix} - \\ - \\ - \end{bmatrix} & 3 \times 1 \end{bmatrix} \\ & \text{Vector: } \begin{bmatrix} w^T \cdot a \\ b \end{bmatrix} = \begin{bmatrix} 2 \times 3 & 2 \times 1 \end{bmatrix} = \underline{\underline{2 \times 1}} \\ & (\underline{\underline{2 \times 1}}) + (\underline{\underline{2 \times 1}}) \end{aligned}$$

$$z' = (w')^T \cdot a + b'$$

$$a' = \sigma(z')$$

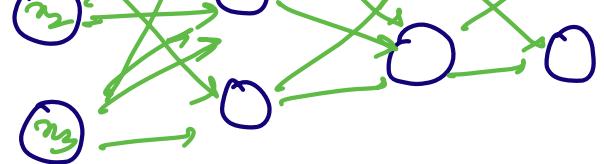


$$z^2 = (\omega^2)^T \cdot a^1 + b^2$$

$$a^2 = \sigma(z^2)$$

$$z^3 = (\omega^3)^T \cdot a^2 + b^3$$

$$\hat{y} = \text{Softmax}(z^3)$$



σ :

- ReLU
- Sigmoid
- Softmax

[1, 2]

$$\frac{e^1}{e^1+e^2}, \frac{e^2}{e^1+e^2}$$

3 neurons [1, 2, 3]

O/p layer:

$$\frac{e^1}{e^1+e^2+e^3}, \frac{e^2}{e^1+e^2+e^3},$$

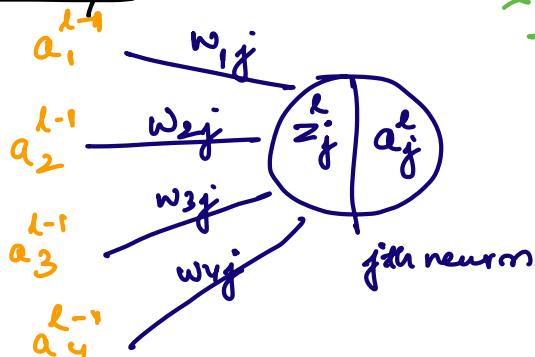
$$\frac{e^3}{e^1+e^2+e^3}$$

Backpropagation:

Loss function:

$$L = (y_{\text{actual}} - y_{\text{predicted}})^2$$

Output Layer:



L = last layer / output layer

Binary classification task:

Binary Cross Entropy

$$Y = (a - \sigma)^2$$

$$z_j^l = \sum_i w_{ij}^l a_i^{l-1} + b_j^l$$

$$\frac{\partial L}{\partial w} = ?$$

$$\frac{\partial L}{\partial b} = ?$$

$w_{ij} \rightarrow z_j \rightarrow a_j \rightarrow \text{Loss}$

$$\frac{\partial L}{\partial w_{ij}} = \frac{\partial L}{\partial a_j} \cdot \frac{\partial a_j}{\partial z_j} \cdot \frac{\partial z_j}{\partial w_{ij}}$$

$$\frac{\partial L}{\partial a_j} ?$$

$$L = \frac{1}{2} \sum_i (y_i^l - a_i^l)^2$$

\downarrow
i ranges over the no. of neurons
in o/p layer

$$\frac{\partial L}{\partial a_j} = \frac{1}{2} \cdot 2 (y_j - a_j) (-1)$$

$$\frac{\partial L}{\partial a_j} = -(y_j - a_j)$$

$$\frac{\partial a_j}{\partial z_j} ?$$

$$a_j = \sigma(z_j)$$

$$\frac{\partial a_j}{\partial z_j} = \sigma'(z_j)$$

$$\frac{\partial a_j}{\partial z_j} = \sigma(z_j)(1 - \sigma(z_j))$$

$$\begin{array}{c} \text{U} \\ x? y \text{ is min?} \\ \frac{\partial y}{\partial x} \end{array}$$

$w? L \text{ is min?}$

o/p by us

$$a = 0.3$$

$$a = 0.7$$

Dog

$$\text{Loss}_{\text{dog}} = (-0.3)^2 + (0.7)^2$$

$$\text{Loss} = \frac{(y - a)^2}{2}$$

$$\begin{array}{c} \text{TD:} \\ \hline 1 \\ 0 \end{array}$$

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\frac{\partial \sigma(z)}{\partial z} = \frac{\partial (1 + e^{-z})^{-1}}{\partial z}$$

$$= \frac{-1 (e^{-z})(-1)}{(1 + e^{-z})^2}$$

$$= \frac{e^{-z}}{1 + e^{-z}}$$

$$\frac{\partial z_j}{\partial w_{ij}} ?$$

$$\begin{aligned}
 &= \left(\frac{1}{1+e^{-z}} \right) \left(1 - \frac{1}{1+e^{-z}} \right) \\
 &= \sigma(z) (1 - \sigma(z))
 \end{aligned}$$

$$z_j^l = \sum_i w_{ij}^l a_i^{l-1} + b_j^l$$

$$\frac{\partial z_j}{\partial w_{ij}} = a_i^{l-1}$$

Combine:

$$\frac{\partial L}{\partial w_{ij}} = \underbrace{\frac{\partial L}{\partial a_j}}_{\text{red circle}} \cdot \underbrace{\frac{\partial a_j}{\partial z_j}}_{\text{red circle}} \cdot \underbrace{\frac{\partial z_j}{\partial w_{ij}}}_{\text{red circle}}$$

$$\frac{\partial L}{\partial w_{ij}} = \frac{-(y_j - a_j) \sigma(z_j) (1 - \sigma(z_j))}{\sigma_j^2} a_i^{l-1}$$

$$\frac{\partial L}{\partial w_{ij}} = \varepsilon_j^l a_i^{l-1}$$

$$\varepsilon_j^l = -(y_j - a_j) \sigma(z_j) (1 - \sigma(z_j))$$

Bias:

$$\begin{aligned}
 \frac{\partial L}{\partial b_j} &= \underbrace{\frac{\partial L}{\partial a_j}}_{\text{green bracket}} \cdot \underbrace{\frac{\partial a_j}{\partial z_j}}_{\text{green bracket}} \cdot \underbrace{\frac{\partial z_j}{\partial b_j}}_{\text{green bracket}} \\
 &= \varepsilon_j^l
 \end{aligned}$$

$$z_j^l = \sum_i w_{ij} a_i^{l-1} + b_j^l$$

$$\frac{\partial z_j}{\partial b_j} = 1$$

$$\frac{\partial L}{\partial b_j} = \varepsilon_j^l \cdot 1$$

Output Layer:

$$\frac{\partial L}{\partial w_{ij}} = \delta_j^L a_i^{l-1}$$

$$\delta_j^L = -(y_j - a_j) \sigma(z_j)(1 - \sigma(z_j))$$

$$\frac{\partial L}{\partial b_j} = \delta_j^L \cdot 1$$